Appendix with supplementary information

Supplementary information and documentation for "Risk Is Not Symmetric: An Intraday Study of the Bond Market Term Premia Around Macroeconomic Announcements". Not to be included in final version.

A Data description

The futures contract contain a number of underlying futures contract with differing expiry and settlement dates. The first contract is called the front futures contract and is the contract closest to expiry. The subsequent 3 futures contracts, called the 1^{st}, 2^{nd} and 3^{rd} back contracts are the contracts respectively with the second shortest expiry, third shortest expiry and 4th shortest expiry.

The price of the Eurodollar futures contract is determined as $P_t = 100 - R_t^{i,i+3}$, where $R_t^{i,i+3}$ is the expected 3-month rate interest rate at expiry in i months. In our case, we use $i = 0, 3, 6, 9$, that is in respectively 3, 6, 9 and 12 months from today. From the price the monetary policy path at delivery dates for the futures contract is easily calculated. Specifically, the monetary policy path at the 12-month horizon is given by[1]

$$r_t^{12M} = \sum_{i=0,3,6,9} \log(1 + \frac{R_t^{i,i+3}}{100})$$

The use of interest rate futures contract data is however problematic in one minor sense. The interest rates considered, are 3-month forward rates with fixed delivery dates. Firstly, this implies that the actual levels become distorted by the cost-of-carry on these futures contracts. Secondly, the 12-month rates are also forward 12-month dates, but with differing days to

[1] In principle, a longer monetary policy path length may be chosen, such as an 18-month horizon. Data availability prevented us from using a longer period.

forward start. The procedure used in this paper is similar to that used in Faust, Rogers, Wang, and Wright (2007)[2]. They, like us, look only at intraday changes, rather than levels. The cost-of-carry argument will consequently not be a problem, as cost-of-carry changes only at a day-to-day level and similarly the small differences in differing forward start days only gives very small measurement errors (second order effects) compared to using actual 3-month rates. The gains of having access to liquid intraday developments however by far exceed the very minor inaccuracies of using futures data instead of actual money market rates.

[2]The working paper version of their paper has an elaborate description of the data, which is similar to our data.

B A model for decomposing money market rates

We start by defining our affine term structure model (AFTM), see also Ang and Piazzesi (2003), Duffie and Kan (1996), Dai and Singleton (2002) and Cambell, Lo, and MacKinlay (1997). The price of a zero coupon bond at time t with time-to-maturity n in an AFTM is given by:

$$P_t^n = \exp\left[A_n + \mathbf{B}_n^{'}\mathbf{X}_t\right] \tag{1}$$

$\mathbf{X}_t$ denotes a vector of state variables in the economy

$$\mathbf{X}_{t+1} = \boldsymbol{\mu} + \boldsymbol{\rho}\mathbf{X}_t + \boldsymbol{\Sigma}\boldsymbol{\varepsilon}_{t+1}$$

in which $\boldsymbol{\rho}$ is the autoregressive parameter matrix, $\boldsymbol{\mu}$ is its vector of constants, and $\boldsymbol{\Sigma}$ denotes the covariance matrix for the underlying shocks in the economy, $\boldsymbol{\varepsilon}_{t+1}$, specified to be homoscedastic. The coefficient A_n and the matrix $\mathbf{B}_n$ only depends upon the *maturity* of the bond, and respect the following recursions visualizing the no-arbitrage restrictions imposed upon the financial markets by the AFTM:

$$\begin{aligned} A_{n+1} &= A_n + \mathbf{B}_n^{'}\left(\boldsymbol{\mu} - \boldsymbol{\Sigma}\boldsymbol{\lambda}_0\right) + \frac{1}{2}\mathbf{B}_n^{'}\boldsymbol{\Sigma}\boldsymbol{\Sigma}^{'}\mathbf{B}_n - \delta_0 \\ \mathbf{B}_{n+1}^{'} &= \mathbf{B}_n^{'}\left(\boldsymbol{\rho} - \boldsymbol{\Sigma}\boldsymbol{\lambda}_1\right) - \boldsymbol{\delta}_1^{'} \end{aligned} \tag{2}$$

The yield of a bond which matures in the next period must equal risk free rate which in the AFTM follows:

$$i_t = \delta_0 + \boldsymbol{\delta}_1^{'} \mathbf{X}_t$$

The vector $\boldsymbol{\delta}_1^{'}$ determines the loading of the state variables in the economy to the risk free rate of interest, while δ_0 determines the level of the risk free rate of interest in the absence of any shocks.

Our task is to decompose the futures into risk premia and expectations and the key determinants behind risk premia are the parameters $\boldsymbol{\lambda}_0$ and $\boldsymbol{\lambda}_1$. This can be seen by the model implied excess holding period return:

$$\begin{aligned} E_t\left[hpr_{t+1}^n\right] &\equiv E_t\left[p_{t+1}^{n-1} - p_t^n\right] - i_t \\ &= -\frac{1}{2}Var_t\left(hpr_{t+1}^n\right) - cov_t\left(m_{t+1}, hpr_{t+1}^n\right) \qquad (3) \\ &= -\frac{1}{2}\mathbf{B}_{n-1}^{'}\boldsymbol{\Sigma}\boldsymbol{\Sigma}^{'}\mathbf{B}_{n-1} + \mathbf{B}_{n-1}^{'}\boldsymbol{\Sigma}\boldsymbol{\Lambda}_t \qquad (4) \end{aligned}$$

The first term is a Jensen inequality term while the second term is a risk premium, which arises from a non-zero covariance between the discount factor and the return on the asset. From (4), the functional form for the risk premia in an AFTM is given by $\mathbf{B}_{n-1}^{'}\boldsymbol{\Sigma}\boldsymbol{\Lambda}_t$: $\boldsymbol{\Lambda}_t \equiv \boldsymbol{\lambda}_0 + \boldsymbol{\lambda}_1^{'}\mathbf{X}_t$ denotes the *market price of risk*; the Sharpe ratio that an asset must earn if it loads on a specific shock.[3]

[3] The Sharpe ratio is defined as the excess return above the risk free rate divided by its standard deviation. It can be shown that the standard deviation of (4) in an AFTM equals $B_{n-1}^T\Sigma$. Disregarding the Jensen-term, Λ_t therefore equals the Sharp ratio.

The market price of risk is also equal to the Girsanov kernel used to change probability measure, so $\mathbf{A}_n, \mathbf{B}_n$ are functions of the stochastic processes for the state variables in the economy under the equivalent martingale measure. $\mathbf{B}_{n-1}^{'}$ is the loading on bond prices of a shock to the state-variables in the economy such that $\mathbf{B}_{n-1}^{'}\mathbf{\Sigma}$ together is the quantity of risk or the expected fluctuation which investors can expect from bond prices.

B.1 Estimation of the affine yield curve model

We need to estimate the parameter vector, $\Theta \equiv \left(\delta_0, \boldsymbol{\delta}_1^{'}, \mathbf{\Sigma}, \boldsymbol{\mu}, \boldsymbol{\rho}, \mathbf{\Lambda}_t\right)$, for the AFTM. We follow Chen and Scott (1993) and use a one-step maximum likelihood estimation, which is both a first best econometric methodology and a feasible way to estimate the parameter vector in the model. The first task in this methodology is to determine the number of latent factors sufficient to price the money market rates. Using a standard principal component analysis, the first and second latent factor explain 0.9952 per cent of the variation in the bond yields and we consequently use only a 1-factor model.[4] Having 4 yields with different maturities we assume the 6-month and 12-month money market rates are measured without error, while the 3-month and 9-month money market rates are assumed to be measured with error. Equation (1)

[4] Standard in the literature is to use a 3-factor model, see Dai and Singleton (2000) and Ang and Piazzesi (2003). This paper, however, estimates money market rates for the very short end of the yield curve only. Further, the second, third, and forth principal component only explain 0.0047 per cent, 0.0001 per cent, and 0 per cent respectively of the total variation in the money market rates.

can thus be inverted for the state-variables, $\mathbf{X}_t$, and the parameter vector, Θ, can be estimated by maximum likelihood. The details can be found in Chen and Scott (1993) or Ang and Piazzesi (2003). Table 1 gives the estimates of the model parameters.

Factor Structure	$X_{t+1} = \mu + \rho_1 X_t + \sigma \varepsilon_t$
ρ	0.9559 (0.0089)
μ	0.0019 (0.0145)
σ	0.0008 (0.0002)
Short Rate	$i_t = \delta_0 + \delta_1 X_t$
δ_0	0.0046 (0.0105)
δ_1	1 (-)
Market Prices of Risk	$\Lambda_t = \lambda_0 + \lambda_1 X_t$
λ_0	0 (-)
λ_1	-5.1950 (0.0050)

Table 1: Estimates for AFTM. The table reports parameter estimates and standard errors in parenthesis for a one-factor affine yield curve model.

It is however widely known that finding the maximum of the likelihood function can be tricky as the function is likely to be quite flat and/or contains local maximum. Starting values thus become very important. We search for the global maximum by random starting values on *daily* data, as even one estimation becomes extremely time consuming when using more than 200.000 observations. We take the estimates for the estimation of the daily data as starting values for the estimation of the intra-daily data.

References

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