Fixed Income Basics

Corporate Finance and Incentives

Lars Jul Overby

Department of Economics
University of Copenhagen

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A few practical issues

- Monday classes moved to 1.1.18.
- Mentor programme
  - See http://alumni.ku.dk/mentorprogram
- What to read?
- Homepage

What is a bond

- A bond is essentially a loan.
- The issuer of a bond raises money directly on capital markets, instead of going through a lender (a bank)
- It has a known cashflow - its payment profile
- Many types of bonds exist (GT, page 45)
  - Zero coupon bonds
  - Bullet bonds
  - Annuity bonds
  - Variable rate bonds
  - Perpetuities

Limitations

We will only consider bonds with a certain cashflow, i.e. we assume that there is no default probability, no embedded options and no variable-rate bonds

- Mortgage bonds: carry imbedded options, which makes them quite complex
- Corporate bonds: here the default probability is most often positive, but of varying size - credit risk
  - What we do cover, however, is a required foundation for any study of those
- Furthermore, we will not talk about what the interest rate should be per se: arguments from macro are needed for this

Debt financing

- The financial environment consists of many other assets than equity.
- One of the major financial assets is bonds.
- The government can borrow money by issuing government bonds.
- The government and central banks also have a role in keeping financial markets flexible by issuing bonds and bills of varying maturity.
- Corporations can also borrow money by issuing corporate bonds.
- Corporations borrow money at a premium compared to the government rate.
- Companies have many different debt instruments they can choose between.

Valuing bonds

Bonds are, in many ways, much simpler to price than equity.

- We know the future cash flows - in nominel terms
- We are in a world of certainty: in this model, there are several points in time, but we know exactly what will happen at each of them
- The fixed income asset generates a vector of cash flows

$$C_i = (c_{i1}, c_{i2}, ..., c_{iT})$$

- The bond market consists of N assets, creating a $N \times T$ matrix of cash flows

$$C = \begin{bmatrix} c_{11} & c_{12} & . & c_{1T} \\ c_{21} & c_{22} & . & c_{2T} \\ . & . & . & . \\ c_{N1} & c_{N2} & . & c_{NT} \end{bmatrix}$$

Valuing bonds

How do we price bonds?

- We find the present value of the future cash flows.
- The price of such assets has to reflect the time-value of money, but not uncertainty in the value of future cash flows

$$\pi_i = C_i d$$

- Seems somewhat unrealistic, but in reality, it works perfectly for government bonds: no developed country is going to default, so if you buy a bond from the Danish government, you are going to get the cash flow that it promises

Assumptions

Again, we assume that markets are frictionless, meaning that

- Investors can go long and short all bonds in any quantity
  - So it is possible to e.g. short 1/6 of bond i, meaning that at time 0 the investor gets $\pi_i/6$, and then pay $c_{it}/6$ at time t for $t = 1, ..., T$
- There are no transaction costs

No arbitrage

- We are going to use no arbitrage arguments to find relationships that must hold between different assets (bonds)
  - Positive payoff implies positive price
- Again, the no arbitrage equilibrium argument is extremely powerful, since a violation means that an infinite amount of money can and will be earned by all investors regardless of preferences
- This means that, for instance, an asset that pays $(3, \ldots, 3)$ should cost more than one that pays $(1, \ldots, 1)$
- And that an asset or portfolio that has nonnegative cash flow at all times must have a nonnegative price
- The key insight is that portfolios can be used to synthetically replicate assets – therefore, to avoid arbitrage, the assets must cost the same as these portfolios

No arbitrage

- Note that the no arbitrage assumption is not very strict: we can create markets that look quite crazy and are still arbitrage free: for instance, if $d_t > 1$, it means that 1 kr invested now becomes worth less than 1 kr at time t
- In reality, this can not occur, since here we can always "stick the money under the mattress", where 1 kr now becomes 1 kr at time t
- This means, that in reality, we would have $d_t < d_{t-1}$ for all t

Complete markets

- An investor can buy any payment stream he desires - either through straight assets or by creating the payment stream synthetically through portfolios of assets

$$C^T \theta = y$$

where θ is an $N \times 1$ vector of portfolio weights and y is an $T \times 1$ vector of cash flows

- This requires there to be at least as many assets as cash flow dates in the economy $(N \geq T)$
- In completet markets, no arbitrage implies that there exists a unique, strictly positive vector of discount factors d
- The market can be described by

$$\pi = Cd$$

Zero coupon bonds

- From now on we assume that the market is arbitrage free and complete – and for simplicity, we also assume $N = T$
- A zero coupon bond (zcb) is the most basic instrument in a bond market:
- The $t-$zcb pays 1 kr at time t and nothing else
- The value (or price) of a zero-coupon bond with a unit payoff at time t is just d_t - the discountfactor

Short rate and spot rates

Assume we have a bond paying 1 kr at time 1. The price at time 0 must be the present value of this payment:

$$\pi_1 = \frac{1}{(1+r_1)} = d_1$$
$$r_1 = \frac{1}{d_1} - 1$$

r_1 is todays one-period spot rate or the short rate (also termed r_0 in L&P). Assume we have a loan paying 1 kr at time 2. The price at time 0 must be the present value of this payment:

$$\pi_2 = \frac{1}{(1+r_2)^2} = d_2$$
$$r_2 = \sqrt{\frac{1}{d_2}} - 1$$

r_2 is todays two-period spot rate.

The spot rates are also known as the yield to maturity on zero coupon bonds

$$y(0,t) = \left(\frac{1}{d_t}\right)^{\frac{1}{t}} - 1$$

The term structure of interest rates is given by the series of spot rates or YTM on zero-coupon bonds

To find out what these spot rates are, we can look in the market to find bonds which make a single certain payment.

- zero-coupon bonds
  - In the US we have stripped bonds or strips, which are bonds that only payout one cash flow at expiration.
  - In Denmark, we only have such bonds for short maturities i.e. up to 12 months - skatkammerbeviser.
- bonds with only one payment remaining
- synthetic bonds with only one payment

Creating ZCB's

Even though we don't have zero-coupon bonds for every maturity, we can still derive the term structure if we have complete markets.

- We create synthetic zero-coupon bonds
- Or bootstrapping

Bootstrapping

Find a portfolio weight vector θ which results in portfolio cash flows like those af a ZCB

$$C^T\theta = e_t$$

where e_t is a vector paying of 1 unit at time t and 0 at all other times
The price of this portfolio will be

$$\pi^T\theta = (Cd)^T\theta = d^T C^T\theta = d^T e_t = d_t$$

Bootstrapping

Assume we have a ZCB that pays of 1 at $t = 1$ and a bond which pays of 1 at $t = 1, 2$. Also assume we know the price of these two bonds. How do we find the discount rates?

$$
\begin{aligned}
\pi_1 &= \frac{1}{1+r_1} = d_1 \\
\pi_2 &= \frac{1}{1+r_1} + \frac{1}{(1+r_2)^2} = d_1 + \frac{1}{(1+r_2)^2} = d_1 + d_2 \\
d_2 &= \pi_2 - d_1
\end{aligned}
$$

Basically the same as creating a synthetic portfolio consisting of long 1 unit of the two period bond and short 1 of the ZCB.

In reality

- In reality, there are many more points in time than there are bonds: technically, there is a point in time for every day for all eternity (or at least for the maturity of the longest running bond, i.e. around 30 years)
- But only around 20 Danish government bonds nominated in DKK (as well as a few in EUR) are traded
- This means that in reality, markets are incomplete: makes good sense, since it seems unlikely that it should be possible to use traded bonds to construct a portfolio that pays 1 kr on July 5, 2014, and nothing on any other date
- For bootstrapping purposes, this means that $Cd = \pi$ has several solutions
- So in practice, to find a d vector, a certain functional form of d_t is assumed, with just a few parameters, and then the parameters are chosen to make d_t fit as good as possible
- It fits quite nicely!

Yield to maturity

The yield to maturity is the single discount rate, which we can use to discount all the payments on our bond and get the same π as if we discounted each payment by the relevant spot rate.

$$\begin{aligned} \pi &= \frac{c_1}{(1+r_1)} + \frac{c_2}{(1+r_2)^2} \\ &= \frac{c_1}{(1+y)} + \frac{c_2}{(1+y)^2} \end{aligned}$$

The internal rate of return on the bond:

$$\pi = \Sigma_{i=1}^{T} \frac{c_i}{(1+y)^i}$$

Term structure and bond yields

- The yield to maturity on a bond depends on the timing of the bonds cash flows.
- If we have an upward sloping term structure, bonds making relatively small payments in the short term have higher YTM than bonds making relatively high payments in the short term.
- i.e. bonds with higher coupon payments, all else equal, have lower yields than bonds with small coupons - if the term structure is upward sloping.
- If we have an downward sloping term structure, bonds making relatively small payments in the short term have lower YTM than bonds making relatively high payments in the short term.
- i.e. bonds with large coupon payments, all else equal, have higher yields than bonds with small coupons - if the term structure is downward sloping.