

# Exercise Set 7 - Real Investments and Real Options\*

Corporate Finance and Incentives - Fall 2010

## Problem 1

The common stock in a company paid an annual dividend yesterday of \$22.5 and has a beta of 1.2. The expected return to the market is 12% and the risk free rate is 7%.

- a) What is the fair value of the stock if dividends are expected not to grow in the future?
- b) What is the fair value of the common stock if dividends are expected to grow at 2% a year?

## Problem 2

Imagine that we have a company, where all future revenues are independent of the chosen capital structure, assume that corporate debt is risk free and finally assume, that we have no taxes.

- a) Find a relation relating asset-beta, the beta to the stock and the beta of the debt of this company.
- b) Find a relation relating the expected return to all assets in the company, the expected return to the common stock and debt of this company.
- c) What is likely to happen, if the company takes on a massive amount of debt?
- d) Define the concept of WACC in a world where there are no corporate taxes.
- e) If a company has a WACC of 12% and the risk free rate is 6%, will adopting a project with an internal rate of return of 8% create value.

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\*Compiled by Jacob Lundbeck Serup; September 2006. Last edited by Carsten S. Nielsen; November 2010.

### Problem 3

A company has to decide whether to build 20 apartments for a total cost of \$80 mio. One year after the initial investments have been made, the project is done and the market value of the apartments depend on the state of the world. The risk free rate is 5% and the return to the market is also dependent on the state of the world.

- a) Assuming that the CAPM is correct, use the "Risk Adjusted Discount Rate Method" to find the time zero value of the project.
- b) Use the "Certainty Equivalent Method" to find the value of the real estate project.
- c) Are the results the same and if not, explain why they might differ.
- d) Which method is in general to be preferred?
- e) Can a tracking portfolio be constructed in this case?

| State | Probability | Market Return | Value of Apartments |
|-------|-------------|---------------|---------------------|
| 1     | 0.33        | 0.26          | 108                 |
| 2     | 0.33        | 0.14          | 100                 |
| 3     | 0.34        | -0.10         | 72                  |

### Problem 4

Now we assume that we are in a world with corporate tax rate of  $T_c$  but there are no taxes on personal income.

- a) Explain why the inclusion of corporate taxes might favor using debt as a means for corporations to raise capital.
- b) Relate this phenomenon to the Modigliani-Miller Theorem
- c) Draw a balance sheet including the debt tax shield and the unlevered assets using the model of Hamada.
- d) Find the relationship between the beta of the assets, the beta of the unlevered assets and the beta of the tax shield.
- e) Find the relationship between the return to the assets, the return to the unlevered assets and the return to the debt tax shield.
- f) Fill out the of the table shown below assuming a tax rate of  $T_c = 1/3$ ,  $D + E = 1$ , and debt is risk-free.

| $\beta_A$ | $\beta_E$ | $\beta_{UA}$ | $D/E$ | $D$ | $E$ |
|-----------|-----------|--------------|-------|-----|-----|
| -         | 1.20      | -            | -     | 0.3 | -   |
| -         | -         | 0.4          | 2     | -   | -   |

### Problem 5

The Danish government knows that there is oil on Greenland but unfortunately it is not easily accessible, so whether to extract it or not depends highly on the oil price. The oil price will evolve according to the diagram shown below and there is a risk free rate of 4%. Imagine that it is possible to extract 1 mil. barrels of oil each year for the next two years at a cost of \$35 a barrel.

- a) Find the price of the oil using risk neutral valuation.
- b) Can a replicating portfolio be constructed and what does it consist of?

Imagine that the Danish government decides to extract all the oil and sell it on the forward market immediately.

- c) Find the forward price of oil one year and two years from now.
- d) What is the value of the oil in Greenland using this strategy?

Now we see what happen if the Danish government can shut down production of oil in some of the nodes.

- e) When is it optimal to shut down production?
- f) Assume that the forward market is not used and find the value of the oil using risk neutral valuation.
- g) Can the Danish government do even better, if they have the possibility to shut down production and use the forward market?

