

Exercise Set 4 - Arbitrage Pricing Theory*

Corporate Finance and Incentives - Fall 2008

Problem 1

- Express the APT in two ways. First using the factors themselves, then using factor risk premia.
- The APT framework relies on the assumption that we can construct so called pure factor portfolios. Explain what a pure factor portfolio is and what conditions for the error terms must be met for us to be able to apply the no-arbitrage argument.
- Explain how we can replicate any asset using pure factor portfolios and use a no-arbitrage argument to justify the APT.
- In general there are two ways of pricing securities. We either use a market equilibrium argument or a no-arbitrage argument. Describe the relationship between the CAPM and the APT and explain which argument underlies each of the models.
- What is the minimum number of factors needed in order to explain the expected returns of a group of 10 securities, if the securities have no firm specific risk?

Problem 2

Consider the following two factor model for the returns of three stocks. Assume that the factors and ϵ_i have mean 0, that all factors have a variance of 0.01 and are uncorrelated, and that $\sigma^2[\epsilon_a] = 0.01$, $\sigma^2[\epsilon_b] = 0.04$ and $\sigma^2[\epsilon_c] = 0.02$.

$$r_a = 0.13 + 6F_1 + 4F_2 + \epsilon_a$$

$$r_b = 0.15 + 2F_1 + 2F_2 + \epsilon_b$$

$$r_c = 0.07 + 5F_1 - 1F_2 + \epsilon_c$$

- What are the expected returns of the assets.
- Find the variance-covariance matrix of the return to the three assets.
- Construct one portfolio with $\beta_1 = 0$, $\beta_2 = 1$ and a portfolio with $\beta_1 = 1$, $\beta_2 = 0$.

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d Compute the expected return and risk premia of these two portfolios.

e Finally introduce a fourth asset with the following factor equation.

$$r_d = 0.15 + F_1 + F_2 \tag{1}$$

Does this give rise to an arbitrage opportunity, if we assume that the APT holds, and if so, how can we exploit this?

Problem 3

Consider a financial market with three assets, whose returns are given by.

$$\begin{bmatrix} r_1 \\ r_2 \\ r_3 \end{bmatrix} = \begin{bmatrix} 0.275 \\ 0.400 \\ 0.675 \end{bmatrix} + \begin{bmatrix} 0.50 & 0.25 \\ 0.50 & 0.50 \\ 0.75 & 1.00 \end{bmatrix} \cdot \begin{bmatrix} F_1 \\ F_2 \end{bmatrix}$$

where F_1, F_2 are independent and normally distributed random variables with mean 0 and variance 1.

a Construct a portfolio of these three assets, which gives a risk-free return.

b Now assume that in addition to the original assets there exists a risk-free asset with $r_f > 0$. For which value of r_0 is the exact APT fulfilled, and what are the factor risk premia in this case?

c Finally assume that $r_f = 0.05$. How can an investor use, that APT is not satisfied to earn money with no risk?