Exercise Set 3 - Capital Asset Pricing Model[*]

Corporate Finance and Incentives - Fall 2010

Problem 1

Assume that AOL, Microsoft and Intel have the variance-covariance matrix and expected returns shown below. Furthermore assume that the risk free rate is 5%.

a) Find the expected return and variance of the minimum-variance portfolio. Try to give an intuitive explanation as to why the formulas you use, when you find the global minimum-variance portfolio and the market portfolio, are valid.

b) Find the expected return and variance of the tangency portfolio.

Table 1: Data for AOL, Microsoft and Intel

Stock	AOL	Microsoft	Intel	Mean Return
AOL	0.002	0.001	0.000	15%
Microsoft	0.001	0.002	0.001	12%
Intel	0.000	0.001	0.002	10%

c) Find the efficient frontier of risky assets in two different ways and draw it in a diagram. In the same diagram mark the global minimum-variance portfolio, the tangency portfolio and the individual assets.

- Construct the efficient frontier by calculating the weights in the tangency portfolio for different values of the risk free rate.
- Use the Two Fund Separation Theorem to find the efficient frontier from portfolio weighted averages of the minimum-variance portfolio and the tangency portfolio, when the risk free rate is 5%.

d) Show that an equally weighted portfolio of AOL, Microsoft and Intel can be improved upon by marginal-mean, marginal-variance analysis.

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[*]Compiled by Jacob Lundbeck Serup; September 2006. Last edited by Benjamin Falkeborg and Carsten S. Nielsen; October 2010.

e) Argue why the tangency portfolio is the market portfolio, draw the efficient set of *all* assets and the Capital Market Line.

f) Now assume that the world consists of many assets and give the argument, why the covariance between any asset and the market portfolio is the relevant measure of risk for all assets.

g) Explain how the ratio between excess return and covariance with the market for different assets must be related in equilibrium, and use this equilibrium condition to deduce the CAPM.

h) Use the above derivation to find the Beta of AOL, Microsoft and Intel

i) Find the Beta of an equally weighted portfolio of AOL, Microsoft and Intel.

Problem 2

Consider a market with two risky assets A and B. All market participants expect asset A to give a return of 10% and asset B to give a return of 30%. Furthermore they know that the variance of asset A is 0.01, that the variance of asset B is 0.09 and that the assets are uncorrelated. Finally there is also a risk-free asset in the economy with a return of 9%.

a) Find an explicit expression for the efficient frontier of risky assets.

b) Find the composition of the global minimum variance portfolio, and calculate it's expected return and variance.

c) Find the the composition and variance of the one mean-variance efficient portfolio with an expected return equal to the risk-free rate.

d) Find the market portfolio of this economy.

Problem 3

Since you would like to invest some money in two stocks A and B, you have hired an investment manager to give you some advice. The manager has insured you that the CAPM holds, and that the following is true:

i. A portfolio (portfolio 1) with 50% invested in stock A, 30% in stock B and the rest in the risk less asset has an expected return of 14.64%, a beta of 1.08 and a variance of 0.0333

ii. Another portfolio (portfolio 2), which consists of 20% stock A, 70% stock B and the rest in the risk-free asset, has an expected return of 12.24% and a beta of 0.78.

iii. Covariance between the return to portfolio 1 and portfolio 2 has been estimated to 0.02724.

iv. The standard deviation of the market return is 0.12.

v. The correlation coefficient between the return to stock A and the return to the market portfolio is 0.72.

a) Find the risk-free return and the return to the market portfolio and draw the capital market and the security market line in the appropriate diagrams.

b) Find the beta and expected return to the two stocks A and B

c) Find the variance-covariance matrix of assets A and B.

d) Find the expected return and the standard deviation for the five portfolios given in the table below. Draw these portfolios in a mean-variance diagram and combine them to construct the efficient frontier of stock A and B.

Table 2: Portfolio Weights

Portfolio	3	4	5	6	7
Stock A	100%	75%	50%	25%	0%
Stock B	0%	25%	50%	75%	100&