Exercise Set 1 - Fixed Income Securities*

Corporate Finance and Incentives - Fall 2010

Problem 1

Consider a financial market with N assets and T different states (points in time). The payoff of the N assets in the different states T is collected in an N by T dimensional matrix denoted C. The prices of the assets are collected in a vector π and we denote a vector of portfolio weights by θ. Finally we denote a vector of discount factors for different points in time by d.

a) State the price and payoff vector of a portfolio with weights θ in terms of C, π and θ.

b) State the conditions under which the markets are complete.

c) Define arbitrage of type one and two and state the conditions under which the market is arbitrage free.

$$C = \begin{pmatrix} 10 & 15 & 25 \\ 40 & 30 & 20 \\ 30 & 30 & 35 \end{pmatrix}, \pi = \begin{pmatrix} 20 \\ 80 \\ 55 \end{pmatrix}$$

$$C = \begin{pmatrix} -5 & -5 & -105 \\ 12 & 12 & 12 \\ 10 & 20 & 30 \end{pmatrix}, \pi = \begin{pmatrix} -98 \\ 10 \\ -40 \end{pmatrix}$$

$$C = \begin{pmatrix} 0 & 0 & 45 \\ -20 & -20 & -20 \\ 38 & 36 & 34 \end{pmatrix}, \pi = \begin{pmatrix} 35 \\ -52 \\ 94 \end{pmatrix}$$

d) Consider the markets given below and for each market find the price and payoff vector of an equally weighted portfolio.

e) Investigate whether the markets are complete and arbitrage free.

*Compiled by Jacob Lundbeck Serup; September 2006. Last edited by Carsten S. Nielsen; August 2010.

Problem 2

Consider a 2-year government bullet bond with a face value of 100$ paying a coupon of 12%. Coupon payments are made January 1. We assume that each month is treated normally, and that the year has 365 days, regardless of leap year status. The bond initially trades at par, and the term structure is assumed flat.

a) Find the yield to maturity of the bond, when coupons are payed annually, semiannually, quarterly, monthly and continuously. When payments are annual, you can find an analytic solution, but otherwise you might want to use a spread sheet.

Now assume that coupon payments are made semi-annually.

b) Find the accrued interest on the following dates:
   - July 1
   - May 16
   - April 1
   - December 13

c) Find the price of the bond when the annual yield to maturity is:
   - 10.00%
   - 12.65%
   - 11.20%

d) Find the annual yield to maturity of the bond if the price of the bond is:
   - 102.00
   - 97.30
   - 98.45

Problem 3

Consider a financial market with 3 points in time and the following data:

$$C = \begin{pmatrix} 10 & 10 & 110 \\ 40 & 40 & 40 \\ 8 & 108 & 0 \end{pmatrix}, \pi = \begin{pmatrix} 100 \\ 100 \\ 98 \end{pmatrix}$$

The face value of all three bonds is 100.

a) Find the coupon rate for the annuity bond.

b) Is the market complete?

c) Is the market arbitrage free?

d) Find zero coupon bond prices for all three years, the term structure of interest rates and the forward rates $f(0,1)$ and $f(0,2)$.

e) Check that the following formula holds:

$$1 + y(0,t) = [(1 + f(0,0)) \times (1 + f(0,1)) \times \cdots \times (1 + f(0,t-1))]^{\frac{1}{t}}.$$

f) Construct a three year zero coupon bond, and find the price of this portfolio.

Problem 4

Consider a world with four different assets. A bullet and an annuity bond with two years to expiry and a bullet and an annuity bond with four years to expiry. All bonds pay an annual coupon of 6%.

a) Find the cash flows of each bond.

b) Plot the price as a function of the yield to maturity for all four bonds.

c) Define the concepts of duration and convexity and comment on the plot found under a).

Now assume that the term structure of interest rates is flat at 5%.

d) Find the price of the four year bullet bond.

e) Find the duration and convexity of the four year bullet bond.

f) Find the price change in percent for a shift in the interest rate by 10bp (0.1%). Do it in two ways: (i) using only duration, and (ii) using both duration and convexity.

Finally assume that the one, two, three and four year zero coupon bond prices are equal to $(0.9700, 0.9335, 0.8954, 0.8548)$.

g) Find the price of the four year bullet bond.

h) Find the duration and convexity of the four year bullet bond in this case, where the term structure of interest rates is not flat.

Problem 5

Imagine that we are in a world with five different assets. They all have a principal of 100 and pay coupons once a year, starting exactly one year from now. Further information about the five assets is given below:

i) A bullet bond with a coupon rate of 4% a price of 100.97 and 1 year to maturity

ii) A bullet bond with a coupon rate of 5% a price of 102.87 and 2 years to maturity

iii) A bullet bond with a coupon rate of 6% a price of 105.66 and 3 years to maturity

iv) An annuity bond with a coupon rate of 4% a price of 100.06 and 4 years to maturity

v) A serial bond with a coupon rate of 4% a price of 99.68 and 5 years to maturity

a) Find the payoff matrix C.

b) Assume that the overnight rate is 1.8% and draw the term structure of zero coupon interest rates and the one year forward rates in a diagram.

c) Check that:

$$1 + y(0,t) = [(1 + f(0,0)) \times (1 + f(0,1)) \times \cdots \times (1 + f(0,t-1))]^{\frac{1}{t}}.$$

d) Find the yield to maturity of each asset.

e) Find the yield to maturity of an equally weighted portfolio of asset 1, 2 and 3.

e) Explain why the yield to maturity can't be found as the weighted average of the yield to maturity of the underlying assets in the portfolio.

From now on we assume, that the term structure of interest rates is flat at 4%.

g) Find the price of each bond with a flat term structure of interest rates.

h) Try to order the five assets according to their duration and check, if the order you found is true by calculating the actual duration.

i) An investor wishes to construct a portfolio that gives him a guaranteed payoff from a bond portfolio three years from now, even if the term structure of interest rates changes. Do this in two ways:

- Construct a portfolio from asset 2 and 5 with a duration of 3
- Construct a zero coupon bond using assets 1, 2 and 3

j) Explain the problems facing the investor using the first approach.