Exercise Set 6 - Options*

Corporate Finance and Incentives - Fall 2008

Problem 1

a) Draw the payoff diagram of the following financial instruments at expiry T for a strike price of K.

   i. A European call option
   ii. A European put option
   iii. A forward contract

b) Show how a forward can be constructed using a call and a put option. What is this phenomenon called?

*Compiled by Jacob Lundbeck Serup; September 2006. Last edited by Ian Rusu and Carsten S. Nielsen; November 2008.

Problem 2

What view on the market is reflected in each of the following strategies.

a) *Bull Spread:* Buy one European call and sell a second one with the same expiry date but a larger strike price.

b) *Bear Spread:* Buy one European call and sell another one with the same expiry date but a smaller strike price.

c) *Straddle:* Buy one European call and one European put with same strike price and expiry.

d) *Strangle:* Buy one European call and one European put with the same expiry date and different strike prices.

e) *Butterfly spread:* A portfolio of some financial instruments has the payoff shown below. Construct such a payoff using

   i. European calls

   ii. European puts

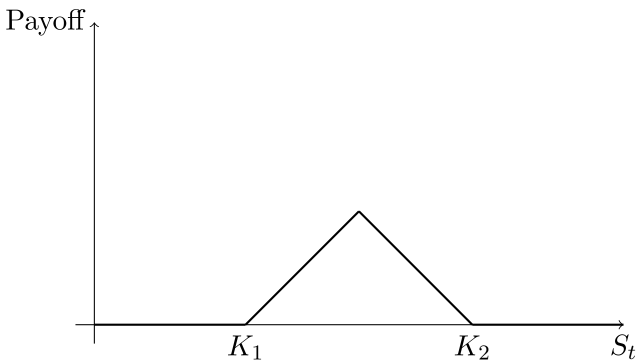


Figure 1: Payoff Diagram for a "Butterfly Spread"

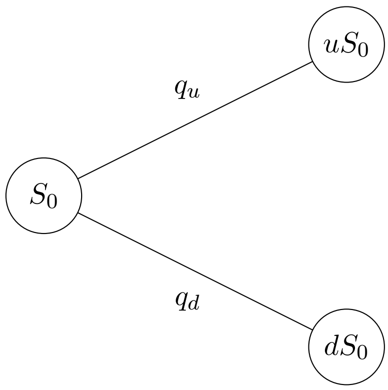
Problem 3

a) When we price options and derivatives, we use the "Equivalent Martingale" or "Risk Neutral Measure". Explain what this implies and in particular what it does *not* imply about investor preferences.

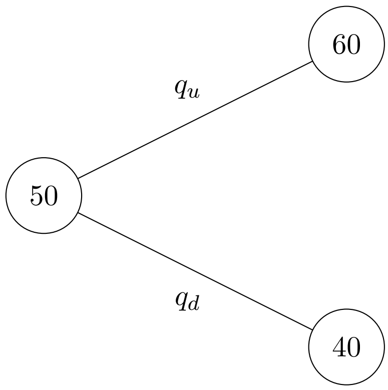
b) What is the rationale behind using the equivalent Martingale measure for pricing options?

c) What can we say about the market if an equivalent Martingale measure exists?

d) Assume that a risk-free asset exists paying a rate of r_f and assume that $d < 1+r_f < u$. Find an equivalent Martingale measure for the payoff diagram shown below.



e) Finally assume that the risk free interest rate is 5% and find the risk neutral probabilities in the diagram shown below, where the numbers are the price of the stock at different times and states.



Problem 4

The price of a stock moves up or down over the next two years as shown in the diagram below. The risk free rate is 5% and the stock does not pay dividends.

a) What is the price and replicating portfolio at each node of a European call option with a strike price of $130?

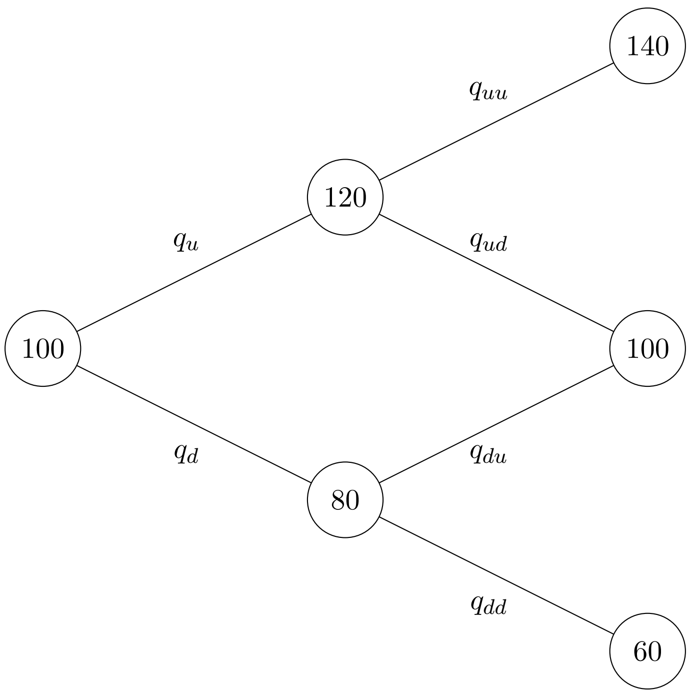
b) What is the price and replicating portfolio at each node of a European put option with a strike price of $130?

c) Use the above to illustrate put-call parity.

d) Would it be optimal to exercise an American put with a strike price of $130 at either the up or the down node?

e) What is the price of the American put option?

f) Several effects come in to play, when the investor decides if he wants to exercise an American put option before expiry, even if the stock does not pay dividends. Explain these effects and why it might be optimal to exercise an American put option early.



Problem 5

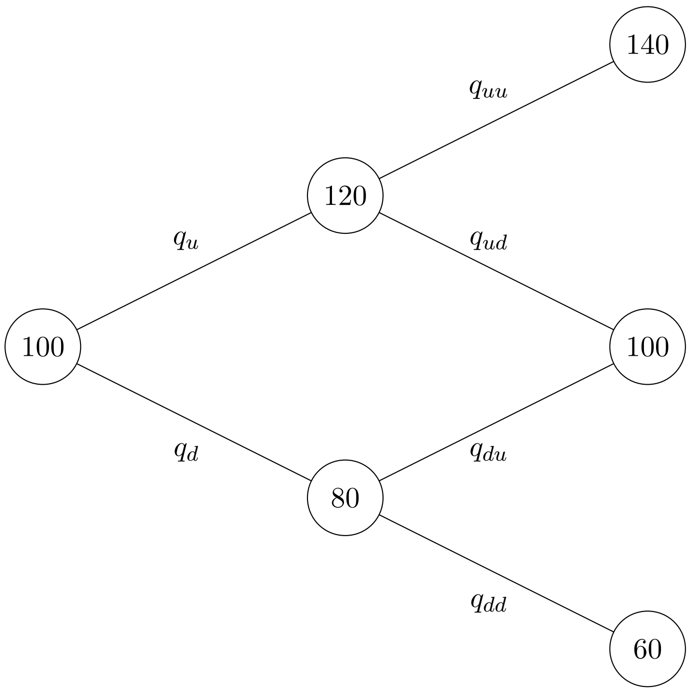
Consider a stock that evolves according to the diagram shown below. The risk free asset pays an interest of 4% and for the first four questions you can assume, that the stock does not pay dividends.

a) Find the risk neutral measure at each node.

b) Find the value of a European call option with a strike price of \$80.

c) Construct a replicating portfolio at each node.

d) Find the price of an American call option with the same strike price. Explain why it is never optimal to exercise an American call option before expiry on a non-dividend paying stock.



Now assume that the Stock pays a dividend of \$10 in the second period. This will change the price of the stock but you can assume, that the percentage change in all subsequncent nodes are the same as before.

e) What is the new price of the stock at t=1 and t=2 in the different nodes? (construct a new three)

f) Find the value and replicating portfolio at each node of a European call with strike price of \$80.

g) Will the investor exercise the option before expiry if the strike price is \$80?

h) Find the value and replicating portfolio at each node of an American call option with a strike price of \$80.

Problem 6

The Black-Scholes formula for the price of a European call option, which matures at time T is given by.

$$c(S_0) = S_0\phi(d_1) - Ke^{-rT}\phi(d_2)$$

$$d_1 = \frac{\ln\left(\frac{S_0}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \qquad d_2 = \frac{\ln\left(\frac{S_0}{K}\right) + \left(r - \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$$

a) State the conditions under which the Black Scholes formula is derived.

b) Find $\phi(-\infty)$, $\phi(0)$ and $\phi(\infty)$.

c) Try to give an intuitive interpretation of the formula and $\phi(d_1)$ and $\phi(d_2)$.

d) Use Put-call parity to find the Black-Scholes formula for a European put option.

e) Define the concept of implied volatility

Imagine that we are in a world, where the Black-Scholes model is valid. There exist a risk-free asset with an annual return of 5% and a stock with a volatility of 20%.

f) Find d_1, d_2, $\phi(d_1)$, $\phi(d_2)$ and the price c_0 of a European call option with 0.5 years to maturity, a strike of $K = 100$ when the price of the underlying asset is 95.

g) Find $-d_1$, $-d_2$, $\phi(-d_1)$, $\phi(-d_2)$ and the price p_0 of a European put option with 0.5 years to maturity, a strike of $K = 100$ when the price of the underlying asset is 95.